04 - Percentages - Notes

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This set of notes is part of a series of Numerical Reasoning Test (NRT) preparation resources which you can find at www.numericalreasoningtestsuccess.com.

These resources are organised into a number of different topics. For each topic, there is a set of notes (such as this one) and a question pack.

Each set of notes explains a set of skills, with example questions for each one. Within the question pack for the same topic, you can find practice questions (with answers) for each of these skills.

I advise that you work through the sets of notes in order. Within each set of notes, start by reading the explanation of the first skill. Then go to that skill in the question pack and complete the practice questions. Only once you have mastered a skill should you move onto the next one. And only once you have mastered all the skills in a set of notes should you move on to the next set of notes. This approach is called *mastery learning*.

If you find any errors in this document (including mathematical errors, typos or any other mistakes), please let me know at contact@numericalreasoningtestsuccess.com.

Convert between percentages and decimals.

Examples

1) What is 64% as a decimal?

2) Convert 0.793 to a percentage.

3) What is 0.6% as a decimal?

Notes

Converting between percentages and decimals on a calculator is very straightforward:

- To convert a percentage to a decimal, divide it by 100.
- To convert a decimal to a percentage, multiply it by 100.

To do Example 1, we need to divide 64 by 100:

64 ÷ 100 = 0.64

Therefore the answer is 0.64.

To do Example 2, we need to multiply 0.793 by 100:

0.793 x 100 = 79.3

Therefore the answer is 79.3%. The calculator will not show a percentage sign on the output - it does not know we are calculating a percentage. We add this because we know that what we are finding is a percentage.

To do Example 3, we need to divide 0.6 by 100:

0.6 ÷ 100 = 0.006

Therefore the answer is 0.006.

Once you have practiced a few of these, you may find that you can do them in your head more quickly than you can on the calculator. If so, then it may be a good idea to do so. However, it is easy to make silly mistakes when doing this in your head - if in doubt, use the calculator. The most reliable method would be to do it in your head and then check it on the calculator, but this is obviously quite time consuming. As you practice answering questions, you will develop the approach that works best for you.

Convert between percentages and fractions.

Examples

1) What is 36.8% as a fraction?

2) What is $\frac{3}{14}$ as a percentage?

3) Convert 0.2% to a fraction.

Notes

We have already learnt how to convert between fractions and decimals (using the S⇔D button on the calculator) and how to convert between decimals and percentages (by dividing or multiplying by 100). We can combine these skills to convert between percentages and fractions:

- To convert a percentage to a fraction, divide it by 100. Most calculators will then display the fraction. If the calculator displays it as a decimal, use the S⇔D button to convert it to a fraction.
- To convert a fraction to a percentage, multiply it by 100. If the calculator shows this as a fraction, use the S⇔D button to show the percentage.

To do Example 1, we type in 36.8 ÷ 100 and press =. The calculator displays the fraction, which is $\frac{46}{126}$ (if the calculator shows the decimal, we use S⇔D to get the fraction).

To do Example 2, we type in 3 ÷ 14 x 100 (or if you prefer to use the fraction button, type in $\frac{3}{14}$ x 100) and then press =. The calculator displays this as a fraction ($\frac{150}{7}$). We press S⇔D and the calculator displays the percentage, which is 21.42857143% (the calculator does not add the percentage symbol - we do this because we know it is a percentage).

To do Example 3, we type in 0.2 ÷ 100 and press =. The calculator displays the fraction, which is $\frac{1}{500}$(if the calculator shows the decimal, we use S⇔D to get the fraction).

Find a given percentage of a given amount.

Examples

1) What is 14% of 5,764,655?

2) Find 135% of 62.

3) What is 75% of 900?

Notes

To find a percentage of an amount, we multiply the percentage by the amount. For example, 30% of 73 is equal to 30% x 73.

However, most scientific calculators do not have a percentage button. Therefore, you must first convert the percentage to a decimal or fraction and then multiply that by the amount.

So, to find 30% of 73, we could do 0.3 x 73 or we could do $\frac{3}{10}$x 73 - both of which give the answer: 21.9.

To do Example 1, we need to first convert 14% to a decimal or fraction.

You may be able to quickly divide 14 by 100 in your head, which would tell you that 14% as a decimal is 0.14. In which case you would simply type in 0.14 x 5,764,655 and press =.

If you weren't confident doing that conversion in your head, all you would need to do is type in 14 ÷ 100 x 5,764,655 and press =. Division and multiplication have equal priority, so the calculator works from left to right, first dividing 14 by 100, which converts the 14% to a decimal (or fraction), and then multiplying this by 5,764,655.

Either way, you get the answer, which is 807,051.7.

To do Example 2, we type in 1.35 x 62 (or, if we aren't confident with the conversion, 135 ÷ 100 x 62) and press =. This gives the answer, which is 83.7.

To do Example 3, we type in 0.75 x 900 (or, if we aren't confident with the conversion, 75 ÷ 100 x 900) and press =. This gives the answer, which is 675.

Alternatively, we may recognise that 75% is equal to $\frac{3}{4}$. If we spot this, then we could do 3 ÷ 4 x 900 (or, if you like using the fraction button,$\frac{3}{4}$x 900), which also gives the correct answer.

It is only really worth using a fraction conversion like this if it is extremely obvious to you and you are completely confident of it. The type of situation where it can be most useful is when the top number of the fraction we convert to is 1. For example, suppose we had

been asked to find 25% of 900. 25% is equal to $\frac{1}{4}$, and multiplying by $\frac{1}{4}$is the same thing as dividing by 4. Therefore, we could just type in 900 ÷ 4 and press =, which would be a bit quicker than typing in 0.25 x 900 and pressing =. However, as you can see, the time savings are pretty minimal.

Calculate what percentage a given amount is of another given amount.

Examples

1) What percentage of 64 is 48?

2) What is 35 as a percentage of 20?

3) What percentage of 500 is 2?

Notes

We have already learnt how to find what fraction a number is of another number. Therefore, to find what percentage a number is of another number, we can just find what fraction it is and then convert that to a percentage.

In Example 1, we type in 48 ÷ 64 and press = (we divide by 64 because the question asks for 48 as a percentage '**of** 64'). The calculator displays the result as a fraction: $\frac{3}{4}$(or your calculator may display it as a decimal: 0.75 - the next steps are the same either way). To convert this to a percentage, we multiply by 100 (by typing in x 100 and pressing =), which gives 75. Therefore, the answer is 75%.

You could also do Example 1 in one step by typing in 48 ÷ 64 x 100 and pressing =. This is the approach we will use for Examples 2 and 3.

Another alternative is just to do 48 ÷ 64 and press = (which gives $\frac{3}{4}$), use S⇔D to convert this to a decimal (which is 0.75) and then mentally convert this to a percentage (75%). Or you could even just mentally convert $\frac{3}{4}$to 75%. As you can see, being confident with doing these types of conversions in your head will save you time, but isn't essential since you can always use the calculator.

To do Example 2, we type in 35 ÷ 20 x 100 and press = (we divide by 20 because the question asks for 35 as a percentage '**of** 20'). This gives 175, so the answer is 175%.

To do Example 3, we type in 2 ÷ 500 x 100 and press =. This gives 0.4, so the answer is 0.4%.

A common mistake here would be to see 0.4 on your calculator display and think that you need to multiply this by 100 to convert it to a percentage, giving 40%. This is incorrect as we have already multiplied by 100. The answer is 0.4%.

Given an amount and what percentage it is of another amount, calculate the other amount.

Examples

1) 66 is 44% of another amount. What is the other amount?

2) 35% of an amount is 196. What is that amount?

3) 102,600 is 160% of another amount. What is the other amount?

Notes

To do this type of question we simply divide the amount we are given by the percentage.

To understand why this works, let's use the letter X to represent the amount we are looking for in Example 1. The question tells us that 66 is 44% of X. Therefore:

X x 44% = 66

To find X, we divide both sides by 44%:

X = 66 ÷ 44%

Most scientific calculators do not have a percentage button, so before we can do this on the calculator, we need to convert the percentage to a decimal or fraction.

You may be able to convert 44% to a decimal in your head. The answer is 0.44. Otherwise you can do 44 ÷ 100 on the calculator. Then we just have to type in 66 ÷ 0.44 (if we just calculated the 0.44 on the calculator, we could type in 66 ÷ Ans) and press =. This gives the answer, which is 150.

If we want to do the conversion and the final calculation in one step, then we need to use brackets, like so:

66 ÷ (44 ÷ 100) =

To do Example 2, we type in 196 ÷ 0.35 and press =. This gives the answer, which is 560.

To do Example 3, we type in 102,600 ÷ 1.6 and press =. This gives the answer, which is 64,125.

Calculate percentage increase or decrease.

Examples

1) The population of the UK in 2001 was 59,113,000. In 2011 it was 63,182,000. What was the percentage increase in the UK population over this time?

2) A company's profits in 2019 were £245,653. In 2020 they were £212,352. What was the percentage decrease in profit between these two years?

3) A school had 876 pupils last academic year. It has 854 this academic year. What is the percentage change? Is this an increase or a decrease?

Notes

Percentage change is the amount that something has changed, expressed as a percentage of its original value. If it has increased, then we call this a percentage increase. If it has decreased, we call it a percentage decrease.

To calculate percentage increase or decrease we need to combine two skills that we have learnt previously. First, we have to work out the difference between the original value and the final value. Then we have to calculate what percentage this is of the original value.

In Example 1, we are asked to calculate the percentage increase in population.

First, we calculate the difference between the original value and the final value by typing in 63,182,000 - 59,113,000 and pressing =. This gives us 4,069,000, which is the amount that the population has increased by.

Then we have to work out what percentage this is of the original value. To do this we need to divide 4,069,000 by the original population, which is 59,113,000 and then multiply by 100 to make it a percentage. We can use Ans for this, since 4,069,000 is the answer to the previous calculation. We type in:

÷ 59,113,000 x 100

The calculator fills in the Ans for us. It displays:

Ans ÷ 59,113,000 x 100

Then we press =. This gives 6.883426657. Therefore, the population has increased by 6.883426657%.

In Example 2, we are asked to find the percentage decrease. The method is essentially the same.

First, we find the difference:

245,653 - 212,352 =

This gives 33,301, which is the amount the profit has decreased by.

Then we find what percentage this is of the original profit:

÷ 245,653 x 100 =

This gives 13.55611371. So the profit has decreased by 13.55611371%.

In Example 3, we are asked to calculate the percentage change and then asked whether it is an increase or a decrease. We can immediately see that it is a decrease by just looking at the numbers in the question: the number of pupils has gone down from 876 to 854. We calculate the percentage decrease following the same steps as above:

876 - 854 =

This gives 22.

÷ 876 x 100 =

This gives 2.511415525. Therefore, the number of pupils has decreased by 2.511415525%.

Given the initial value and the percentage increase or decrease, calculate the final value.

Examples

1) Between the years 2000 and 2015, the price of a Freddo chocolate bar rose by 150%. If the price of a Freddo in 2000 was 10p, what was the price in 2015?

2) The number of visitors to an aquarium was 117,824 in 2019. If there was a 75% decrease in visitors between 2019 and 2020, how many visitors were there in 2020?

3) The global atmospheric CO_2 concentration in January 2011 was 391 parts per million. By January 2021, this had increased by 6.14%. What was the global atmospheric CO_2 concentration (in parts per million) in January 2021?

Notes

There are multiple ways to approach this type of question. We will look at two of them.

The first is to calculate how much the given percentage of the initial amount is, and then adjust the initial amount by this much.

In Example 1, we know that the initial amount was 10p and there was a percentage increase of 150%. So, we start by finding 150% of 10p:

1.5 x 10 =

This gives us 15p. If you aren't comfortable converting 150% to a decimal (1.5) in your head, you could alternatively do 150 ÷ 100 x 10 =.

Since it is a percentage *increase*, we need to add the amount of the increase - 15p - to the initial amount - 10p. Because 15 is the answer to the previous calculation, we can take advantage of Ans by just typing in:

+ 10 =

The calculator will then add 15 and 10, giving 25. Therefore, the price of a Freddo in 2015 was 25p.

The second method is a little bit more complicated to think about, but is quicker to do once you get the hang of it. It involves working out what percentage of the initial amount the final amount is, and then finding that percentage of the initial amount.

In Example 1, we know that the price has increased by 150%. The initial price was 100% of itself (every value is 100% of itself, since that is what 100% means). Since it was initially

100% of itself and then it increased by 150% of itself, the final amount is 250% of the initial amount: 100% + 150% = 250%.

Therefore, we just have to find 250% of 10p. So all we have to type into the calculator is:

10 x 2.5 =

This gives us 25, so we know the final price is 25p.

In Example 2, the initial number of visitors was 117,824 and then it decreased by 75% of itself.

Using the first method, we start by finding 75% of 117,824:

0.75 x 117,824 =

This gives 88,368, which is the amount that the number of visitors has decreased by (we could also have done 75 ÷ 100 x 117,824 =).

Since this is a decrease, we need to subtract 88,368 from the initial number of visitors. We can do this using Ans:

117,824 - Ans =

This gives 29,456. Therefore, the number of visitors in 2020 was 29,456.

To use the second method, we start by working out what the final value is as a percentage of the initial value. We know that the initial value was 100% of itself and then it decreased by 75% of itself. Therefore, the final value is 25% of the initial value: 100% - 75% = 25%.

Therefore, we have to find 25% of the initial value:

0.25 x 117,824 =

This gives us 29,456, which is the correct answer.

There is actually an even quicker way to do this particular question. It comes from recognising that 25% is the same as $\frac{1}{4}$, and multiplying by $\frac{1}{4}$ is the same thing as dividing by 4. Therefore, we can just divide the initial amount by 4:

117,824 ÷ 4 =

This gives 29,456, which is correct.

In Example 3, the initial concentration was 391 parts per million and it increased by 6.14%.

Using the first method:

We start by finding 6.14% of 391:

0.0614 x 391 =

This gives 24.007 (we could also have done 6.14 ÷ 100 x 391 =).

Then we add this on to the initial amount:

+ 391 =

This gives 415.0074. Therefore, the global atmospheric CO_2 concentration (in parts per million) in January 2021 was 415.0074 parts per million.

Using the second method:

The amount was initially 100% of itself and then it increased by 6.14% of itself. Therefore, the final amount is 106.14% of the initial amount (100% + 6.14% = 106.14%).

So we find 106.14% of 391:

1.0614 x 391 =

This gives 415.0074, which is the correct answer.

Given the final value and the percentage increase or decrease, calculate the initial value.

Examples

1) If a hotel had 3,048 guests this year, and this was a 60% increase on last year, how many guests did it have last year?

2) The total rainfall in the UK in May 2021 was 495% higher than in April 2021. If the total rainfall in the UK in May 2021 was 119.595mm, what was the total rainfall in the UK in April 2021?

3) The number of books borrowed from a library decreased by 14% between June and July. If the number of books borrowed in July was 301, how many were borrowed in June?

Notes

For this type of question, we need to first work out what the final amount is as a percentage of the initial amount. We then divide the final amount by this percentage to get the initial amount.

In Example 1, the number of guests increased by 60%. Therefore, the final number of guests is 160% of the initial number of guests (100% + 60% = 160%).

We can express this as an equation:

Initial amount x 160% = Final amount

If we divide both sides by 160%, we get the following:

Initial amount = Final amount ÷ 160%

This shows that we can find the initial amount by dividing the final amount by the percentage. In this case, we need to do:

3048 ÷ 1.6 =

This gives 1905. If you aren't confident converting 160% to a decimal (1.6) in your head, you could alternatively do 3048 ÷ (160 ÷ 100) =, which gives the same answer. Either way, we get the answer, which is that the hotel had 1905 guests last year.

In Example 2, the rainfall increased by 495% to 119.595mm. Since it increased by 495%, the final amount must be 595% of the initial amount (100% + 495% = 595%).

Therefore, we need to divide the final amount by 595% (which is 5.95 as a decimal):

119.595 ÷ 5.95 =

This gives us 20.1. Therefore, the total rainfall in the UK in April 2021 was 20.1mm.

In Example 3, the number of books borrowed decreased by 14% to 301. Since it decreased by 14%, the final amount must be 86% of the initial amount (100% - 14% = 86%).

Therefore, we need to divide the final amount by 86% (which is 0.86 as a decimal):

301 ÷ 0.86 =

This gives us 350. Therefore, the number of books borrowed in June was 350.