05 - Ratio and Proportion - Notes

www.numericalreasoningtestsuccess.com

This set of notes is part of a series of Numerical Reasoning Test (NRT) preparation resources which you can find at www.numericalreasoningtestsuccess.com.

These resources are organised into a number of different topics. For each topic, there is a set of notes (such as this one) and a question pack.

Each set of notes explains a set of skills, with example questions for each one. Within the question pack for the same topic, you can find practice questions (with answers) for each of these skills.

I advise that you work through the sets of notes in order. Within each set of notes, start by reading the explanation of the first skill. Then go to that skill in the question pack and complete the practice questions. Only once you have mastered a skill should you move onto the next one. And only once you have mastered all the skills in a set of notes should you move on to the next set of notes. This approach is called *mastery learning*.

If you find any errors in this document (including mathematical errors, typos or any other mistakes), please let me know at contact@numericalreasoningtestsuccess.com.

Given the ratio of two values, express it in its simplest form.

Examples

1) Express the ratio 45:70 in its simplest form.

Notes

A ratio shows the proportion between things. In Example 1, we have the ratio 45:70, which we would pronounce, "*Forty-five to seventy"* - in other words, the colon (:) in the ratio is pronounced as, "*to*". This ratio has two numbers (45 and 70), which means that it shows the proportion between two things. Specifically, it says that for every 45 of the first thing there are 75 of the second thing. For example, it could be a person's ratio of spending to income, in which case it would tell us that for every £45 of spending there is £70 of income.

If you take a ratio and multiply or divide all of the numbers by the same number, the new ratio you get is equivalent to the one you started with.

For example, say we are told that the ratio of lizards to big cats in a zoo is 3:5. Let's multiply both numbers by 4. This gives us the ratio 12:20. The ratios 3:5 and 12:20 are equivalent ratios. In other words, they are the same ratio, just written in two different ways. The ratio 3:5 tells us that for every 3 lizards, there are 5 big cats. The ratio 12:20 tells us that for every 12 lizards, there are 20 big cats. These are just two different ways of saying the same thing.

The phrase ‘simplest form of the ratio’ means the form which uses the smallest possible whole numbers. To find the simplest form, we need to find the largest number that both of the numbers can be divided by (which is called their largest common factor), and then we need to divide both numbers by that.

In Example 1, the ratio we are given is 45:70. The largest common factor of 45 and 70 (the largest number they can both be divided by) is 5. Therefore, we divide both numbers by 5:

45 ÷ 5 = 9

70 ÷ 5 = 14

Therefore, the simplest form of the ratio 45:70 is 9:14.

To do this, we had to work out that the largest common factor of 45 and 70 is 5. However, we can actually avoid this by getting the calculator to do it for us.

The process of finding the largest common factor of the two numbers and then dividing both numbers by that factor is exactly what the calculator does when it simplifies a fraction. Therefore, all we have to do is make a fraction with the first number of the ratio at the top and the second number of the ratio at the bottom, get the calculator to simplify that fraction, and then put the numbers back into a ratio.

So, for Example 1, we would use the calculator to simplify the fraction $\frac{45}{70}$. This gives $\frac{9}{14}$. Then we just put those numbers back into a ratio, which gives us 9:14.

In other words, although the calculator does not have a function that simplifies ratios, it does have the ability to simplify fractions, and since what we do to the numbers is the same in both processes, we can use the calculator’s fraction simplifying ability as a way to get it to simplify ratios for us.

Note that this will not work if the ratio has three or more numbers in it (for example, 15:33:18) because a fraction only has two parts so there is no way to enter all the numbers.

Given the ratio of three or more values, express it in its simplest form.

Examples

1) Express the ratio 84:36:60 in its simplest form.

Notes

As explained previously, in order to express a ratio in its simplest form, we need to find the highest common factor of all of the numbers in the ratio and then divide all of the numbers by that factor.

When the ratio has three or more numbers in it, we cannot use the calculator’s fraction simplifying ability to do the hard work for us. Therefore, the process is a bit more complicated than simplifying a ratio that only has two numbers in it.

We will at three different ways to do it:

The first method is to list out all of the factors of each number and then find the largest factor that they all have in common.

To find all of the factors of a number, you can work your way up from 1, dividing the number you are looking for factors of by each number to see if you get a whole number. If you get a whole number, then the number you divided by and the number you got are both factors. Once the number you are dividing by is greater than or equal to a number already in your factors list, you know that you have finished. (Note: you can speed this process up considerably if you know the rules for checking divisibility).

So in the case of Example 1:

Factors of 84: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

The largest factor that they all have in common is 12. Therefore, 12 is the highest common factor of 84, 36 and 60. So to find the simplest form of the ratio 84:36:60, we need to divide all three numbers by 12:

$84 \div 12 = 7$

$36 \div 12 = 3$

$60 \div 12 = 5$

Therefore, the simplest form of the ratio 84:36:60 is 7:3:5.

The second method is to start by finding any factor that all of the numbers have in common (even if you aren't sure whether it is the largest common factor), divide all of the numbers by that, and then repeat this process until they no longer have any common factors (besides 1).

In Example 1, the ratio we are asked to simplify is 84:36:60. Since they are all even numbers, we know that they are all divisible by 2. Therefore, we could start by dividing them all by 2:

$84 \div 2 = 42$

$36 \div 2 = 18$

$60 \div 2 = 30$

This gives us the ratio 42:18:30. We then check to see if there are common factors. All of the numbers are still even, so we could divide them all by 2 again. However, we might also notice that all of the numbers are in the 6 times table, meaning that they are all divisible by 6. Going with the bigger factor will get us to the answer in fewer steps, so we should divide them all by 6:

$42 \div 6 = 7$

$18 \div 6 = 3$

$30 \div 6 = 5$

This gives us the ratio 7:3:5. We know that these numbers have no factors in common because they are all prime numbers. Therefore, this is the final answer. If they were not all prime numbers, we would need to list out each of their factors and check that they have none in common.

Note, if we had divided all of the numbers by 2 rather than 6 at the previous step, we would have got 21:9:15. We would then have needed to divide all these numbers by 3, which would give us 7:3:5, which is the answer.

The final method is to find the highest common factor by expressing each number as a product of its prime factors. I will not cover how to do this here because it is quite an involved process. However, if you already know how to do it (or are willing to look it up), it can be a relatively quick and reliable way of working out the highest common factor.

Expressing each number as a product of its prime factors gives us:

$84 = 2^2 \times 3 \times 7$

$36 = 2^2 \times 3^2$

$60 = 2^2 \times 3 \times 5$

Therefore, the highest common factor is $2^2 \times 3$, which equals 12.

We then have to divide all of the numbers by 12, as we did in the first method, which gives us the answer - 7:3:5.

Given information about the quantities of different items, express this as a ratio in its simplest form.

Examples

1) In his allotment, Jim grew 66 carrots and 143 tomatoes. What is the ratio of carrots to tomatoes grown by Jim? Give your answer in its simplest form.

2) On Monday, a website had 126 mobile users, 75 desktop users and 33 tablet users. What was Monday's ratio of mobile users to desktop users to tablet users? Give your answer in its simplest form.

Notes

To do this type we question, we have to first express the information we have been given as a ratio and then convert this ratio to its simplest form, using one of the methods we have already covered.

In Example 1, we are told that Jim grew 66 carrots and 143 tomatoes, and we are asked for the ratio of carrots to tomatoes.

We can think of the ratio of carrots to tomatoes as 'Carrots : Tomatoes'. Then we just have to replace the words with the numbers of each item:

Carrots : Tomatoes

66 : 143

So the ratio of carrots to tomatoes is 66:143. All we need to do now is convert this to its simplest form, which we can do using the calculator (with the method we have already learnt). When we do this, we get 6:13, which is the answer.

In Example 2, we are told that there were 126 mobile users, 75 desktop users and 33 tablet users and asked to find the ratio of these values. We follow the same process that we used in Example 1:

Mobile users : Desktop users : Tablet users

126 : 75 : 33

So the ratio is 126:75:33. We now have to convert this to its simplest form using one of the methods we have already learnt. When we do this, we get 42:25:11, which is the answer.

Given how many times larger or smaller one value is than another, work out the ratio of the two values in its simplest form.

Examples

1) A company's sales are 3 times its expenditure. What is the company's ratio of sales to expenditure? Give your answer in its simplest form.

2) The number of geese living on a lake is 5 times less than the number of ducks. What is the ratio of geese to ducks living on the lake? Give your answer in its simplest form.

Notes

To do this type of question, it helps to first write down in words the ratio we are looking for.

In Example 1, we are asked to find the ratio of sales to expenditure. We can write this down as:

Sales : Expenditure

This may not seem necessary, but it can really help us to focus on what we are trying to find and therefore avoid mistakes.

Next, it is helpful to identify which of the two values is smaller. Since the question tells us the sales are 3 times expenditure, we know that expenditure is smaller than sales. The smaller value will always end up being represented by the number 1 in our answer.

Finally, we need to work out how many times larger the bigger value is than the smaller value. In this question, we are told that sales is 3 times expenditure. In other words, for every £1 of expenditure, there are £3 of sales.

Finally, we put all of this information together in our ratio:

Sales : Expenditure

3 : 1

In Example 2, we are told that the number of geese is 5 times less than the number of ducks and asked to find the ratio of geese to ducks.

We start by writing down the ratio we are looking for:

Geese : Ducks

Then, we identify that the number of geese is the smaller number (since it is 5 times *less* than the number of ducks). Therefore, the number of geese will be represented by the number 1 in the ratio.

Finally, we identify that the number of ducks is 5 times more than the number of geese (this follows from the fact that the number of geese is 5 times less than the number of ducks).

Putting this all together, we get our answer:

Geese : Ducks

1 : 5

In a situation with three or more values, given information about how many times larger or smaller different values are than each other, work out the ratio of all the values in its simplest form.

Examples

1) Tina's pocket money is three times as much as Rachel's pocket money. Rachel's pocket money is twice as much as Jon's. What is the ratio of Tina's pocket money to Rachel's pocket money to Jon's pocket money? Give your answer in its simplest form.

2) There are four buildings, A, B, C and D. A is three times taller than B. C is 4 times shorter than B. D is twice as tall as A. What is the ratio of the heights of the four buildings? Give your answer in its simplest form.

Notes

To do this type of question, we start by converting all of the information we have been given into the form of ratios.

In Example 1:

We are told that Tina's pocket money is three times as much as Rachel's. Using what we have learnt previously, we can express this as follows:

Tina : Rachel
3 : 1

We are also told that Rachel's pocket money is twice as much as Jon's. We can express this as follows:

Rachel : John
2 : 1

We need to combine these two separate ratios into one overall ratio for the three people.

It is possible to combine two ratios in this way as long as they have an item in common. In this example, both ratios contain Rachel, therefore they can be combined.

However, before we can combine them, we need to make the number of 'parts' of the ratio that represent the shared item the same in both ratios. So, in this example, we need to make the number of parts that represent Rachel's pocket money the same in both ratios.

At the moment, we have the following:

Tina : Rachel 3 : 1	Rachel : John 2 : 1

In the first ratio, Rachel's pocket money is represented by 1 part. In the second ratio, Rachel's pocket money is represented by 2 parts. Therefore, we need to convert one of the ratios to an equivalent ratio so that the number of parts representing Rachel's pocket money is the same in each case.

The easiest way to do this is to multiply both numbers in the first ratio by 2, so that we go from 3:1 to 6:2. This leaves us with the following:

Tina : Rachel 6 : 2	Rachel : John 2 : 1

We can now combine the ratios into one ratio by simply including all the information, like so:

Tina : Rachel : Jon

6 : 2 : 1

In Example 2, we start by converting all of the information we have been given into ratios. This leaves us with the following:

A : B 3 : 1	B : C 4 : 1	A : D 1 : 2

We need to first combine two of the ratios into one ratio and then combine this ratio with the other remaining ratio. It doesn't matter which order we do this in, so let's work from left to right to keep it simple.

The first two ratios both contain B, which means we can combine them, but first we must make the number of parts for B the same in both ratios. At the moment, in the first ratio B is represented by 1 part and in the second ratio B is represented by 4 parts. Therefore, the simplest next step is to multiply both numbers of the first ratio by 4. That changes the first ratio from 3:1 to 12:4.

So we now have the following:

A : B	B : C	A : D
12 : 4	4 : 1	1 : 2

We can now combine the first two ratios into one:

A : B : C	A : D
12 : 4 : 1	1 : 2

The two ratios this leaves us with both contain A, which means that they can be combined. However, we need to multiply both numbers in the second ratio by 12 so that A is represented by 12 parts in both ratios. This changes the second ratio from 1:2 to 12:24.

So we now have the following:

A : B : C	A : D
12 : 4 : 1	12 : 24

We can now combine these ratios, to get one ratio, which is our final answer:

A : B : C : D
12 : 4 : 1 : 24

Given the ratio of A to B, state the ratio of B to A.

Examples

1) The ratio of guitars to keyboards sold by a music shop is 23:9. What is the ratio of keyboards to guitars sold by the shop?

Notes

The ratio of B:A is just the reverse of the ratio of A:B.

In Example 1, the ratio of guitars to keyboards is 23:9. Therefore, the ratio of keyboards to guitars is 9:23. In other words, you just swap the two numbers in the ratio.

Given the ratio between two or more values, and given one of the values, calculate the other value(s).

Examples

1) The ratio of apples to satsumas to pears in a fruit bowl is 3:5:2. If there are 15 satsumas, how many pears are there?

Notes
In this type of scenario, it is useful to think of the numbers in the ratio as 'parts' of the total. We can then use the information we have been given to work out how much 1 part is equal to, and then scale this up to find the number we have been asked for.

So in this situation, the collection of fruit is made up of 3 parts apples to 5 parts satsumas to 2 parts pears. We have been told that the number of satsumas is 15. Therefore, 5 parts is equal to 15 items:

5 parts = 15 items

To find out how much 1 part is equal to, we therefore need to divide 15 by 5:

1 part = 15 ÷ 5 = 3 items

We now know that 1 part in the ratio is equal to 3 actual items.

We have been asked to find the number of pears. We know from the ratio that 2 parts of the total are pears. Since we have worked out that 1 part is equal to 3 items, we just need to multiply 3 by 2 to work out how much 2 parts is:

2 parts = 3 x 2 = 6 items

Therefore, there are 6 pears in the fruit bowl.

Note that you do not have to write all of this out in full when you are doing these calculations yourself. In fact, all you have to type into the calculator to do this question is:

15 ÷ 5 x 2 =

This will give you the correct answer of 6.

Given the ratio between two or more values, and given one of the values, calculate the total of all of the values.

Examples

1) Alan's pencil case contains black pens, blue pens, red pens and green pens in a ratio of 4:5:2:1 (and there are no other colours of pens in the pencil case). If Alan's pencil case contains 10 blue pens, how many pens does it contain in total?

Notes
As with the previous skill, we have to start by finding out how many items one 'part' in the ratio represents. Then we find out how many parts there are in total by adding together the numbers in the ratio. Finally, we multiply the number of parts by the number of items that one part represents to find the total number of items.

In this question, we are told that there are 10 blue pens and this is represented by 5 parts in the ratio. Therefore, in order to find out how many pens 1 part in the ratio represents, we need to divide 10 by 5:

5 parts = 10 pens

1 part = 10 ÷ 5 = 2 pens

This tells us that 1 part in the ratio represents 2 pens.

Next, we add together all the numbers in the ratio to find the total number of parts:

4 + 5 + 2 + 1 = 12 parts

This tells us that there are 12 parts in total. We have already worked out that 1 part represents 2 pens. Therefore, in order to work out the total number of pens, we need to multiply 12 by 2:

1 part = 2 pens

12 parts = 2 x 12 = 24 pens

Therefore, there are 24 pens in total.

To do this on the calculator we could do:

4 + 5 + 2 + 1 =

And then:

10 ÷ 5 x Ans =

This gives 24, which is the correct answer.

Given the ratio between two or more values, and given the total of all of the values, calculate any given value.

Examples

1) A bike shop sells three types of bikes: mountain bikes, road bikes and hybrid bikes. Last year, the ratio of mountain bikes to road bikes to hybrid bikes sold by the shop was 4:7:3. If the shop sold 112 bikes last year, how many of them were mountain bikes?

Notes

To do this type of question, we first need to add all the numbers in the ratio together to find the total number of 'parts'. Since we have been told how many items there are in total, we can divide this by the number of parts, to find out how many items 1 part in the ratio represents. We can then multiply this by the number of parts for the item we have been asked about in order to find the answer.

In Example 1, we need to start by adding the numbers in the ratio together to find the total number of parts:

4 + 7 + 3 = 14

So there are 14 parts in the ratio in total. We have been told that 112 bikes were sold in total. Therefore, 14 parts in the ratio represents 112 bikes sold. We therefore need to divide 112 by 14 to find out how many bikes 1 part represents:

14 parts = 112 bikes

1 part = 112 ÷ 14 = 8 bikes

We have been asked to find the number of mountain bikes. We know that this is represented by 4 parts in the ratio. Since we know that 1 part represents 8 bikes, we need to multiply 8 by 4 to find out how many bikes 4 parts represents:

1 part = 8 bikes

4 parts = 8 x 4 = 32 bikes

Therefore, 32 mountain bikes were sold.

To do this on the calculator, we could do:

4 + 7 + 3 =

And then:

112 ÷ Ans x 4 =

This gives 32, which is the correct answer.

Given the ratio between two or more items, calculate the fraction that a given item is of the total.

Examples

1) In a school, the ratio of the number of students studying French to the number of students not studying French is 7:4. What fraction of the students are studying French?

Notes

To do this type of question, we have to start by adding together the number in the ratio to find the total number of 'parts'. Since we want to know what fraction a certain item is of the total, we need to find what fraction the number of parts representing that item is of the number of parts representing the total.

In Example 1, we start by finding the total number of parts:

7 + 4 = 11

Therefore, the total number of students is represented by 11 parts.

We know from the ratio that the number of students studying French is represented by 7 parts.

In other words, 7 out of 11 parts of the total student population are studying French. Therefore, we need to find out what fraction 7 is of 11. We have learnt how to do this previously:

$7 \div 11 =$

This gives us the answer, which is $\frac{7}{11}$.

Note that at no point do we need to work out the actual number of students studying French or the actual total number of students. The ratio tells us the proportions between these values, which is all we need.

We could do this whole question in one calculation on the calculator as follows:

$\frac{7}{7+4} =$

Or, alternatively:

$7 \div (7 + 4) =$

Either way, we get $\frac{7}{11}$,which is the correct answer.

In a situation where there are two values, given the fraction or percentage that one value is of the total, find the ratio between the two values, expressed in its simplest form.

Examples

1) A vet surgery only treats cats and dogs. Last year, $\frac{2}{5}$ of the animals treated by the vet surgery were cats. What was the ratio of cats to dogs treated by the vet surgery last year?

2) A bag of marbles contains large marbles and small marbles. If 20% of the marbles are large, what is the ratio of large marbles to small marbles?

Notes

In this type of question, we are told that there are two values and we are told what fraction or percentage one of the values makes up of the total. To work out the ratio between the two values, we simply have to work out what fraction or percentage the other value makes up of the total, and then find the ratio between these two fractions or percentages.

In Example 1, we are told that $\frac{2}{5}$ of the animals are cats. We are also told that there are only cats and dogs. Therefore, the fraction of the animals that are cats and the fraction that are dogs must add up to 1, since they make up all of the animals. Therefore, we can work out what fraction of the animals are dogs by subtracting $\frac{2}{5}$ from 1:

$1 - \frac{2}{5} = \frac{3}{5}$

This shows us that $\frac{3}{5}$ of the animals are dogs.

We now know that $\frac{2}{5}$ of the animals are cats and $\frac{3}{5}$ of the animals are dogs. The ratio of cats to dogs is just the ratio of these two fractions: $\frac{2}{5} : \frac{3}{5}$

However, we need to convert this to its simplest form. We can do this using the usual method on the calculator (it is possible for either of the numbers in a fraction to be fractions). If we do this we get 2:3, which is the correct answer.

Alternatively, we could notice that both of the fractions have the same denominator (bottom number): 5. Therefore, we could just multiply both fractions by 5, which gives us 2:3. We would have to enter this into the calculator to check that it is the simplest form of the ratio, which it is.

In Example 2, we are told that 20% of the marbles are large. Since we are also told that all of the marbles are either large or small, we know that the percentage of marbles that are large and the percentage that are small must add up to 100%, since they make up all of the marbles. Therefore, we can find the percentage of marbles that are small by subtracting 20% from 100%:

> 100% - 20% = 80%
>
> Note: on the calculator, we would convert the percentages to decimals and do 1 - 0.2, which gives 0.8. We would then convert this back to a percentage, which gives 80%.

This tells us that 80% of the marbles are small.

Since 20% of the marbles are large and 80% are small, the ratio of large to small marbles is just the ratio of these percentages: 20% : 80%. We don’t need the percentage signs, since this ratio is equivalent to 20 : 80. All we have to do now is find the simplest form of this. When we do that, we find that it is 1:4.

Given the fraction or percentage that one value is of another, find the ratio between the two values.

Examples

1) The number of goldfish in a pond is $\frac{4}{9}$ of the number of carp. What is the ratio of goldfish to carp in the pond?

2) The number of bowls in a dishwasher is 65% of the number of plates. What is the ratio of bowls to plates in the dishwasher?

Notes

In this type of question, we are told what fraction or percentage one value is of another and then asked to find the ratio between the two values.

Let’s call the two values A and B. We are being asked to find the ratio A:B.

It is very important to realise that we have been told what fraction or percentage A is of B - in other words we have been given the fraction $\frac{A}{B}$ or the percentage that A is of B.

We have **not** been told what fraction or percentage A is of the total (the total is A + B).

When we are given A as a fraction of B, this type of question is extremely simple to do. We know that the fraction we have been given is $\frac{A}{B}$ and the ratio we are looking for is A:B. Therefore, we already have the two numbers of the ratio, we simply have to write them as a ratio. In other words, the first number in the ratio is the top number (numerator) of the fraction and the second number in the ratio is the bottom number (denominator) of the fraction.

In Example 1, we are told that the number of goldfish is $\frac{4}{9}$of the number of carp. Therefore, the ratio of goldfish to carp is 4:9.

Another way to think of it is to realise that the number of carp is 1 of itself (every number is 1 of itself). Since the number of goldfish is $\frac{4}{9}$of the number of carp, the ratio of goldfish to carp is $\frac{4}{9}$: 1. If you convert this ratio to its simplest form you will find that it is 4:9.

When we are given A as a percentage of B, the question is only slightly more complicated. We have to realise that B is 100% of itself (every number is 100% of itself). Since we know what A is as a percentage of B (this is the percentage we are given in the question) and we know what B is as a percentage of B (100%), the ratio of A to B is just the ratio of these two percentages.

In Example 2, we are told that the number of bowls is 65% of the number of plates. We know that the number of plates is 100% of itself. Therefore, the ratio of bowls to plates is 65%:100%, which we can also just write as 65:100. All we have to do then is find the simplest form of this ratio (using our usual method), which gives us 13:20.

Given the ratio between two numbers, express it in the form X:1, where X is a value to be found.

Examples

1) Express the ratio 7:4 in the form X:1, where X is a value to be found.

2) Express the ratio 35:8 in the form X:1, where X is a value to be found.

3) Express the ratio 3:5 in the form X:1, where X is a value to be found.

Notes
When we express a ratio in its simplest form, we find the smallest whole numbers that can be used in the ratio. For example, if we were given the ratio 126:15 we would simplify it to 42:5.

However, this is not always the most useful form of the ratio. Sometimes, we might want to know how much of the first item there is for each one of the second item.

For example, perhaps the ratio of students to teachers on a school trip is 126:15. Knowing that this can be simplified to 42:5 might not actually be that useful. Instead, we might want to know how many students there are per teacher. In that case, we want to find an equivalent ratio in which the second number is 1.

To get from any number to 1, we can divide the number by itself. So, starting from the ratio 42:5, to turn the 5 into a 1, we need to divide it by 5. In order to ensure that the new ratio is equivalent to the original one, we need to do the same thing to both numbers. Therefore, we need to divide both numbers by 5:

42:5

÷5 ÷5

8.4:1

This shows us that the ratio of students to teachers is 8.4:1. In other words, there are 8.4 students for every 1 teacher.

We could have also started from the original ratio of 126:15 and divided both sides by 15. This also gives 8.4:1.

Expressing ratios in the form X:1 is also useful when we want to compare ratios.

For example, say we knew that one school trip had a student to teacher ratio of 42:5 and another school trip had a student to teacher ratio of 191:25. These ratios are both expressed in their simplest form, but they aren't very easy to compare with each other.

If, instead, we convert them to the form X:1, we find that the first school trip had a student to teacher ratio of 8.4:1 and the second trip had a student to teacher ratio of 7.64:1. This is much more useful for comparison: it tells us that the first trip had 8.4 students per teacher and the second had 7.64 students per teacher.

In Example 1, we start with the ratio 7:4. We divide both numbers by 4, which gives us the ratio 1.75:1.

In Example 2, we start with the ratio 35:8. We divide both numbers by 8, which gives us the ratio 4.375:1.

In Example 3, we start with the ratio 3:5. We divide both numbers by 5, which gives us the ratio 0.6:1.

Given a list of ratios, place them in ascending or descending order.

Examples

1) Place the following ratios in descending order: 7:13, 92:39, 3:8, 12:23.

Notes

To place a list of ratios in ascending or descending order, we first have to convert each of them to the form X:1, as we have previously learnt how to do.

In Example 1, this gives us the following (I have rounded each one to 3 sf):

7:13 is equal to 0.538:1

92:39 is equal to 2.36:1

3:8 is equal to 0.375:1

12:23 is equal to 0.522:1

We then place them in ascending or descending order based on the first number of the ratio.

In Example 1, we are asked to place them in descending order, which means most positive to most negative (or biggest to smallest, since they are all positive). This gives us:

92:39 (2.36:1)

7:13 (0.538:1)

12:23 (0.522:1)

3:8 (0.375:1)

Given the ratio between two or more values and the difference between two of the values, calculate all of the values.

Examples

1) Three numbers, A, B and C, have a ratio of 5 : 20 : 13. The difference between A and C is 120. What are the numbers A, B and C?

Notes

In this type of question, we are told the actual difference between two of the values. We are also told the ratio of the values. From this ratio we can work out the difference in number of "parts" of the ratio between the two values that we know the actual difference between. Then, since we know the difference in number of parts between the values, and the actual difference that this represents, we can work out how much one part represents. We can then use this to work out all of the values.

In Example 1, we are told that the actual difference between A and C is 120.

In the ratio, A is represented by 5 parts and C is represented by 13 parts. We need to find the difference between these two numbers:

13 - 5 = 8

This shows us that the difference between A and C, in terms of number of parts of the ratio, is 8 parts. Since we know that the actual difference is 120 parts, we know that 8 parts of the ratio represents an actual value of 120. From here, we can work out how much one part of the ratio represents:

8 parts = 120

÷8 ÷8

1 part = 15

In other words, we divide 120 by 8 and this gives us 15, telling us that 1 part in the ratio represents a value of 15.

We can now work out the values of A, B and C, since we know that they are in a ratio of 5 : 20 : 13.

A is represented by 5 parts in the ratio. Since we know that 1 part represents a value of 15, we need to multiply 5 by 15 to find the value of A:

A = 5 x 15 = 75

We follow the same approach for B and C:

B = 20 x 15 = 300

C = 13 x 15 = 195

So the answer is: A = 75, B = 300, C = 195

If we wanted to, we could check our answer by confirming that the ratio of A to B to C is 5 : 20 : 13 and that the difference between A and C is 120.