06 - Averages - Notes

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This set of notes is part of a series of Numerical Reasoning Test (NRT) preparation resources which you can find at www.numericalreasoningtestsuccess.com.

These resources are organised into a number of different topics. For each topic, there is a set of notes (such as this one) and a question pack.

Each set of notes explains a set of skills, with example questions for each one. Within the question pack for the same topic, you can find practice questions (with answers) for each of these skills.

I advise that you work through the sets of notes in order. Within each set of notes, start by reading the explanation of the first skill. Then go to that skill in the question pack and complete the practice questions. Only once you have mastered a skill should you move onto the next one. And only once you have mastered all the skills in a set of notes should you move on to the next set of notes. This approach is called *mastery learning*.

If you find any errors in this document (including mathematical errors, typos or any other mistakes), please let me know at contact@numericalreasoningtestsuccess.com.

Find the mode of a set of values.

Examples

1) What is the mode of the following numbers? 43, 12, 17, 58, 12, 97, 34, 58, 12

Notes

There are three different types of average that you need to be able to calculate: the mode, the median and the mean.

The mode is simply the most common number (or item) in a list.

One way to find the mode is to make a list of all the numbers and keep a tally of how many times each number comes up.

Applying this to Example 1, we get the following:

Number	How many times it appears
43	I
12	III

17	I
58	II
97	I
34	I

We can see that 12 is the most common number. Therefore, the mode is 12.

Find the median of a set of values.

Examples

1) What is the median of the following numbers? 43, 12, 17, 58, 12, 97, 34, 58, 12

2) What is the median of the following numbers? 13, 54, 73, 16, 28, 4

Notes

To find the median of a set of numbers, we first place the numbers in order (from most negative to most positive), then we find the middle number.

To work out which number is the middle number, we first need to count how many numbers there are. Let's call this n. For example, if there were 7 numbers, we would say that n = 7.

Then we need to work out what (n + 1)/2 equals.

If (n + 1)/2 is a whole number, then the middle number is the number at the (n + 1)/2th position. For example, if n = 7, then (n + 1)/2 = 4. Therefore, the middle number is the 4th number. So the median is the 4th number.

If (n+1)/2 is not a whole number, then we take the two numbers either side of that position, add them together and divide by 2 to get the median. For example, if n = 8, then (n + 1)/2 = 4.5. Therefore, we would take the 4th and 5th numbers, add them together and divide by 2 to get the median.

To do Example 1, we start by writing the list of numbers in order:

12, 12, 12, 17, 34, 43, 58, 58, 97

Next, we count how many numbers there are. There are 9 numbers. We can express this as n = 9.

Next, we find (n + 1)/2, which in this case equals 5 (because (9 + 1)/2 = 10/2 = 5). Therefore, the median is the 5th number, which is 34:

12, 12, 12, 17, [**34**], 43, 58, 58, 97

To do Example 2, we start by writing the list of numbers in order:

4, 13, 16, 28, 54, 73

There are 6 numbers, so n = 6. Therefore (n + 1)/2 = 3.5.

This means that we need to take the 3rd and 4th numbers, which are 16 and 28:

4, 13, [**16, 28**], 54, 73

We add these together and divide by 2:

(16 + 28)/2 = 22

Therefore, the median is 22.

(That last step, adding 16 and 28 together and dividing by 2 is actually finding the mean of 16 and 22, which will become clear below).

Calculate the mean of a set of values.

Examples

1) What is the mean of the following numbers? Give your answer to 3 sf.
   43, 12, 17, 58, 12, 97, 34, 58, 12

Notes

To find the mean of a set of numbers, you add all of the numbers together and divide by the number of numbers.

To do Example 1, we start by adding all of the numbers together:

43 + 12 + 17 + 58 + 12 + 97 + 34 + 58 + 12 = 343

This gives us 343. We then have to divide this by the number of numbers. There are 9 numbers, we need to do 343 divided by 9:

343 ÷ 9 = 38.1 (to 3 sf)

Therefore, the mean of the numbers is 38.1.

Given the mean of a set of values and the number of values, calculate the total of all of the values.

Examples

1) The mean number of books owned by the students in year 11 at a school is 6.8. If there are 315 students in year 11 at the school, what is the total number of books owned by all the students in year 11?

Notes

In this type of question, we are given the mean and the number of numbers and we are asked to find the total of all the numbers.

We can work out how to do this by thinking about how we find the mean. When we find the mean, we use the following process:

Mean = Total of all the numbers ÷ Number of numbers

We can rearrange this to get a formula that will allow us to find the total of all the numbers. To do this, we simply multiply both sides of the equation by ‘Number of numbers’ (I have swapped the two sides of the equation around below to make it easier to interpret):

Total of all the numbers = Mean x Number of numbers

Therefore, in order to find the total of all the numbers, all we have to do is multiply the mean by the number of numbers.

In Example 1, this means that we need to multiply 6.8 by 315:

6.8 x 315 = 2142

Therefore, the total number of books is 2142.

Given the mean of a set of values, and all but one of the values, calculate the missing value.

Examples

1) A primary school has 7 classes: Reception and Years 1 to 6. The number of students in Reception and Years 1 to 5 are as follows: Reception = 33, Yr 1 = 34, Yr 2 = 32, Yr 3 = 34, Yr 4 = 33, Yr 5 = 28. If the mean class size is 32, how many students are in the Year 6 class?

Notes

In this type of question, we are given the mean of a set of values and all but one of the values and we are asked to calculate the missing value.

The first step is to find the total of all the values using the method that we learnt previously (multiply the mean by the number of values). Then, we find the total of the values we have been given (by adding them together). The missing value is equal to the difference between the total of all the values and the total of the values we have been given.

Applying this to Example 1, we start by finding the total of all the values, which in this case means the total number of students in the school:

Total number of students = 32 x 7 = 224

Next, we find the total of the values we have been given. Which in this case means the total number of students in Reception and Years 1 to 5:

Total number of students in Reception + Yr 1-5 =
33 + 34 + 32 + 34 + 33 + 28 =
194

Finally, we find the difference between these two numbers. This tells us the number of students in Year 6:

Number of students in Year 6 = 224 - 194 = 30

Therefore, there are 30 students in the Year 6 class.

Note that we could have done this with slightly fewer button presses. Once we worked out that the total number of students was 224, we could have subtracted all of the numbers we were given from this, like so:

224 - 33 - 34 - 32 - 34 - 33 - 28 = 30

You should use whichever method you are more comfortable with.

Calculate a weighted mean.

Examples

1) A record shop had 567 customers in June with a mean spend of £18.74, 893 customers in July with a mean spend of £23.14, and 725 customers in August with a mean spend of £24.05. What was the mean spend of all the customers in June, July and August?

2) Of all the properties in a town, 50% are houses, 35% are flats and 15% are maisonettes. The mean age of the houses is 97 years. The mean age of the flats is 27 years. The mean age of the maisonettes is 24 years. What is the mean age of all of the properties in the town?

Notes

In this type of question, we are told that the total set of values is made up of several subsets. We are told how many items are in each subset and we are told the mean of each subset. We are asked to calculate the overall mean.

We <u>cannot</u> find the overall mean by simply finding the mean of the means. This is because the number of items is different in each subset. Instead, we need to calculate what is called a *weighted mean*. A weighted mean is a mean that takes into account the number of items in each subset. This is sometimes called the *weight* of each subset. We calculate a weighted mean as follows:

Weighted mean =

((Mean of subset 1 x Weight of subset 1) + (Mean of subset 2 x Weight of subset 2) + etc.) / (Total number of items in all subsets)

The ‘etc.’ in the formula above refers to the fact that the number of subsets will be different for different questions. What you need to do is multiply each subset’s mean by its weight (the number of values) and then add all these values together. Finally, divide by the total number of values (which we can find by adding together the weights of all the subsets).

So, to do Example 1, we do the following:

((£18.74 x 567) + (£23.14 x 893) + (24.05 x 725)) / (567 + 893 + 725)

= £22.30 (to 2 dp)

Therefore, the mean spend for all of the customers in June, July and August was £22.30.

There are lots of different ways we could do this on the calculator. One way is to do it in one step using the fraction button. Note that we don’t need the brackets around each multiplication, because the order of operations means that the multiplications will be done before the additions anyway:

$$\frac{18.74\ x\ 567 + 23.14\ x\ 893 + 24.05\ x\ 725}{567 + 893 + 725} =$$

We can understand why this formula works using what we have learnt previously. When we multiply the mean of each subset by the number of items in the subset (its weight), what we are doing is finding the total of all the values in the subset. Since we do this for all the subsets and then add these values together, what we get is the total of all the values across all of the subsets. When we then divide that by the total number of values in all of the subsets, we get the mean for all of the values.

In order to calculate a weighted mean, we do not need to know the actual number of items in each subset, we simply need to know what proportion the number of items in each subsets makes up of the total number of items.

This is illustrated in Example 2, where we are told what percentage of the properties are houses, flats and maisonettes, but we are not told the actual numbers of houses, flats and maisonettes. In this situation, we can simply use the percentages as our weights, like so (I have added brackets for clarity of communication, but they are not needed):

Mean age = $\frac{(97 \times 50\%) + (27 \times 35\%) + (24 \times 15\%)}{100\%}$

Of course, we cannot type percentages into most calculators. One way to get around this is to simply convert them to decimals (make sure you do this to the 100% on the bottom as well as the percentages on the top):

Mean age = $\frac{(97 \times 0.5) + (27 \times 0.35) + (24 \times 0.15)}{1} = 61.55$

Therefore, the average age of the properties is 61.55 years.

Note that we do not actually need to divide by 1, because any number divided by one is itself. Therefore, all we actually need to type into the calculator is:

97 x 0.5 + 27 x 0.35 + 24 x 0.15 =

Therefore, as long as you are confident in converting percentages to decimals in your head, this is a very quick way to do this type of question.

Alternatively, because all that matters are the proportions between the weights, we could use the percentages and just ditch the percentage symbols:

Mean age = $\frac{(97 \times 50) + (27 \times 35) + (24 \times 15)}{100} = 61.55$

Note that we cannot treat percentages in this way in other situations! It only works here because all that matters is the proportions between the weights, not the weights themselves. 35% is not the same thing as the number 35. It is actually the same thing as the number 0.35. In other words, the weights we are using here are 100 times more than the percentages they represent. But because of the nature of this particular calculation, all of those factors of 100 cancel out, leaving us with the same answer we would have got if we had converted the percentages to decimals. It is fine to use this method as long as you remember that you cannot do this with percentages in general.

(As an aside, if you have ever calculated relative atomic mass in chemistry, you probably used this method).

Given the mean of a set of values, the total number of values, the means of one or more subsets of the values, and the number of values in each subset, calculate the total and mean of the remaining values.

Examples

1) A company has 528 employees, working in three different cities: London, Manchester and Edinburgh. The mean salary of all the employees is £29,587.

There are 248 employees in London and their mean salary is £32,456. There are 106 employees in Manchester and their mean salary is £27,983. What is the total salary of all the employees in Edinburgh and what is their mean salary?

Notes

The steps to do this type of question are as follows:

- Calculate the total of all the values (mean x number of values).
- Calculate the total for each subset that you know the mean and number of values for.
- Subtract these totals from the overall total. This gives you the total for the remaining values.
- To find the mean of the remaining values, divide their total by the number of values (you can find the number of remaining values by subtracting the number of values in each subset from the total number of values).

Applying this to Example 1, we do the following:

First we find the total salary of all the employees of the company (in other words, the total amount the company pays to all of its employees in a year). We do this using the method we have learnt previously, that is multiplying the mean by the number of values:

Total salary of all employees = £29,587 x 528 = £15,621,936

Next, we find the total salary of all the employees in London and the total salary of all the employees in Manchester, using the same approach:

Total salary of London employees = £32,456 x 248 = £8,049,088

Total salary of Manchester employees = £27,983 x 106 = £2,966,198

We can now find the total salary of the Edinburgh employees by subtracting the total salaries of the London and Manchester employees from the overall total salary:

Total salary of Edinburgh employees =

£15,621,936 - £8,049,088 - £2,966,198 =

£4,606,650

Therefore, the total salary of all the employees in Edinburgh is £4,606,650. This answers the first part of the question.

Next, we need to work out how many employees there are in Edinburgh. To do this, we subtract the numbers of employees in London and Manchester from the total number of employees:

Number of employees in Edinburgh = 528 - 248 - 106 = 174

Therefore, there are 174 employees in Edinburgh.

Finally, we find the mean salary of the employees in Edinburgh by dividing their total salary by the number of employees:

Mean salary of Edinburgh employees = £4,606,650 ÷ 174 = £26,475

Therefore, the mean salary of the employees in Edinburgh is £26,475.